

Name: \_\_\_\_\_

CC Algebra 2 Midterm Review #1

- 1) Use any method to find the product.

a)  $(2x^2 - 3x + 5)(x - 2)$

$$\begin{array}{r} 2x^2 \quad -3x \quad 5 \\ \times \quad 2x^3 \quad -3x^2 \quad 5x \\ \hline -4x^2 \quad 6x \quad -10 \end{array}$$

$$2x^3 - 7x^2 + 11x - 10$$

b)  $(xy - 3)^3$

$$\begin{array}{r} xy \quad -3 \\ x^2y^2 \quad -3xy \\ \hline -3xy \quad 9 \end{array}$$

$$x^2y^2 - 6xy + 9$$

$$\begin{array}{r} x^2y^2 - 6xy \quad 9 \\ x^3y^3 \quad -6x^2y \quad 9xy \\ \hline -3x^2y \quad 18xy \quad -27 \end{array}$$

$$x^3y^3 - 9x^2y^2 + 27xy - 27$$

- 2) a. Use long division to divide:  $(-x^3 + 3x^2 + x) \div (x - 2)$

- b. Is  $(x - 2)$  a factor of the polynomial?

$$\begin{array}{r} -x^2 + x + 3 \\ x - 2 \overline{) -x^3 + 3x^2 + x + 0} \\ \underline{+x^3 + 2x^2} \downarrow \\ x^2 + x \downarrow \\ \underline{-x^2 - 2x} \\ 3x + 0 \\ \underline{-3x - 6} \end{array}$$

NO, there is a remainder of 6

- 3) Simplify the expression using only positive exponents

$$\frac{7x^{-3}y^9}{(2x^4y^{-6})^{-2}}$$

$$(7x^{-3}y^9)(2x^4y^{-6})^2$$

$$(7x^{-3}y^9)(4x^8y^{-12})$$

$$28x^5y^{-3}$$

$$\boxed{\frac{28x^5}{y^3}}$$

- 4) Express the following with a rational denominator:

$$\frac{4}{3+\sqrt{2}} \cdot \frac{(3-\sqrt{2})}{(3-\sqrt{2})}$$

$$\frac{12-4\sqrt{2}}{9-2} = \boxed{\frac{12-4\sqrt{2}}{7}}$$

- 5) Express in simplest radical form:  $\sqrt{245x^3y^7}$

$$\sqrt{49x^2y^6} \sqrt{5xy}$$

$$\boxed{7xy^3\sqrt{5xy}}$$

6) Factor each expression completely:

a)  $3m^4 - 75$

$3(m^4 - 25)$

$\boxed{3(m^2+5)(m^2-5)}$

b)  $72 + 18x - 2x^2$

$-2x^2 + 18x + 72$

$-2(x^2 - 9x - 36)$

$\boxed{-2(x-12)(x+3)}$

c)  $20x^2 - 14x - 24$

$2(10x^2 - 7x - 12)$

$2(10x^2 - 15x + 8x - 12)$

$2(5x(2x-3) + 4(2x-3))$

$\boxed{2(5x+4)(2x-3)}$

f)  $9x^2y^2 - 18xy + 8$

$9x^2y^2 - \underline{6xy} - \underline{12xy} + 8$

$3xy(3xy-2) - 4(3xy-2)$

$\boxed{(3xy-4)(3xy-2)}$

i)  $3y^4 + 9y^2 - 6y^3 - 18y$

$3y(y^3 + 3y - 2y^2 - 6)$

$3y(y(y^2+3) - 2(y^2+3))$

$\boxed{3y(y-2)(y^2+3)}$

7) Find the product of the following, in simplest a + bi form.

$-6 + \sqrt{-49}$  and  $2 - \sqrt{-81}$

$-6 + 7i$

$2 - 9i$

$\boxed{51 + 68i}$

2	-9i
-12	54i
14i	-163i <sup>2</sup>
103	

8) Simplify the expression:  $i^{100} + i^{101} + i^{102}$

divide by 4,

use the  
remainder

$i^0 + i^1 + i^2$

$1 + i + (-1)$

$\boxed{i}$

9) Find the product in simplest a + bi form:  $(6 + 2i)(4 - 3i)$

4	-3i	
2i	24	-18i
2i	8i	-6i <sup>2</sup>
6		6

$\boxed{30 - 10i}$

10) If  $g(x) = x^2 - 4x + 3$ , find the value(s) of  $x$  if  $g(x) = 0$ .  $y=0$

$$0 = x^2 - 4x + 3$$

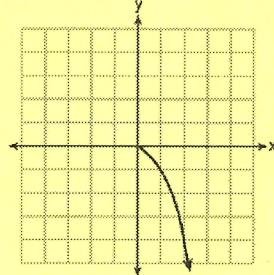
$$0 = (x-3)(x-1)$$

$$\boxed{x=3 \quad x=1}$$

11) Determine the domain and range of the given graph.

$$D: [0, \infty)$$

$$R: (-\infty, 0]$$



12) If  $f(x) = x^2$  and  $g(x) = 2x + 1$ , then determine  $f(g(3x))$ .

$$g(3x) = 2(3x) + 1$$

$$= 6x + 1$$

$$f(6x+1) = (6x+1)^2$$

$$= \boxed{36x^2 + 12x + 1}$$

13) Given function tables on the right, find each:

a)  $f(g(3))$

$$g(3) = 4$$

$$f(4) = \boxed{6}$$

b)  $g(g(4))$

$$g(4) = 6$$

$$g(6) = \boxed{10}$$

$x$	1	2	3	4	5
$f(x)$	3	4	5	6	7

$x$	3	4	5	6	7
$g(x)$	4	6	8	10	12

14) Find the domain of each given function below:

a)  $f(x) = \frac{-3}{x^2 - 1}$

$$x^2 - 1 \neq 0$$

$$(x+1)(x-1) \neq 0$$

$$\boxed{x \neq -1, 1}$$

$$2x + 8 \geq 0$$

$$2x \geq -8$$

$$\boxed{x \geq -4}$$

c)  $h(x) = \frac{1}{\sqrt{5-x}}$

$$5 - x > 0$$

$$-x > -5$$

$$\boxed{x < 5}$$

15) Find the inverse of the following functions:

a.  $f(x) = \sqrt{x-4}$

$$x = \sqrt{y-4}$$

$$x^2 = y - 4$$

$$\boxed{f^{-1}(x) = x^2 + 4}$$

b.  $g(x) = \frac{x}{x+1}$

$$x = \frac{y}{y+1}$$

$$\begin{aligned} xy + x &= y \\ xy - y &= -x \end{aligned}$$

$$\frac{y(x-1)}{(x-1)} = \frac{-x}{(x-1)}$$

$$\boxed{g^{-1}(x) = \frac{-x}{(x-1)}}$$

16) Using the tables on the grid, find the average rate of change on the interval [9, 12].

<b>x</b>	1	3.8	4.7	9	13.8	12
<b>y</b>	3	5.1	8.7	15.8	25.1	30.86

$$\frac{30.86 - 15.8}{12 - 9} = \frac{15.06}{3} = \boxed{5.02}$$

17) Find the average rate of change for each function below:

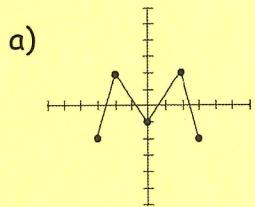
$$f(x) = x^2 - 4x - 12 \text{ on } [-1, 7]$$

$$f(-1) = (-1)^2 - 4(-1) - 12 = -7$$

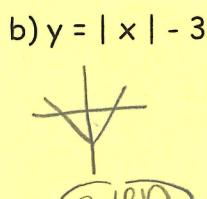
$$\frac{9 - -7}{7 - -1} = \frac{16}{8} = \boxed{2}$$

$$f(7) = (7)^2 - 4(7) - 12 = 9$$

18) Determine if each is even, odd, or neither. Justify your reasoning.



**even**  
(symmetric to  
y-axis)



**even**  
(symmetric  
to y-axis)

$$y = (-x)^2 - 3 \\ = x^2 - 3$$

**even**  
(all signs  
stay)

$$d) f(x) = -3x^3 + 2x^2$$

$$f(-x) = -3(-x)^3 + 2(-x)^2 \\ = 3x^3 + 2x^2$$

**neither**  
(one sign stays  
one sign changes)

$$19) \text{Find the sum in simplest radical form: } \sqrt{5x} + 7\sqrt{80x} + 2\sqrt{180x}$$

$$\sqrt{5x} + 7\sqrt{16 \cdot 5x} + 2\sqrt{36 \cdot 5x}$$

$$\sqrt{5x} + 28\sqrt{5x} + 12\sqrt{5x}$$

$$\boxed{41\sqrt{5x}}$$

20) Solve the equation using any method, and show in simplest radical form.

$$x^2 + 4 = -12x$$

$$x^2 + 12x + 4 = 0$$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-12 \pm \sqrt{128}}{2} = \frac{-12 \pm 8\sqrt{2}}{2}$$

$$\boxed{x = -6 \pm 4\sqrt{2}}$$

21) Solve the equation and express the roots in simplest  $a + bi$  form:

$$x(x - 8) = -17$$

$$x^2 - 8x + 17 = 0$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(17)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{-4}}{2}$$

$$x = \frac{8 \pm 2i}{2}$$

$$x = 4 \pm i$$

22) Find the solution set, algebraically:  $2x^2 = y - 2x - 7$

$$2x^2 = x + 10 - 2x - 7$$

$$10 = y - x \rightarrow y = x + 10$$

$$(-1.5, 8.5)$$

$$2x^2 = 3 - x$$

$$2x^2 + x - 3 = 0$$

$$2x^2 - 2x + 3x - 3 = 0$$

$$2x(x-1) + 3(x-1) = 0$$

$$(2x+3)(x-1) = 0 \quad x = -1.5 \quad x = 1$$

Plug in  $x$

$$x = -1.5$$

$$10 = y - (-1.5)$$

$$10 = y + 1.5$$

$$8.5 = y$$

$$x = 1$$

$$(1, 11)$$

$$10 = y - 1$$

$$11 = y$$

$$x^2 + (1-x)^2 = 61$$

$$x^2 + 1 - 2x + x^2 = 61$$

$$2x^2 - 2x - 60 = 0$$

$$2(x^2 - x - 30) = 0$$

$$2(x-6)(x+5) = 0$$

$$x = 6 \quad x = -5$$

$$x + y = 1 \rightarrow y = 1 - x$$

$$x^2 + y^2 = 61$$

$$(6, -5)$$

$$(-5, 6)$$

Plug in  $x$

$$x = 6$$

$$6 + y = 1$$

$$y = -5$$

$$x = -5$$

$$-5 + y = 1$$

$$y = 6$$

isolate the radical term

25) Find the solution set:  $\sqrt{x+4} + 2 = x$

$$(\sqrt{x+4})^2 = (x-2)^2$$

$$x+4 = x^2 - 4x + 4$$

$$0 = x^2 - 5x$$

$$0 = x(x-5)$$

$$\cancel{x \neq 0} \boxed{x=5}$$

check

$$\cancel{x=0}$$

$$\sqrt{4} + 2 \neq 0$$

$$\cancel{x=5}$$

$$\sqrt{5+4} + 2 = 5$$

$$3 + 2 = 5 \checkmark$$

26) Write the equation of the quadratic given that one root is  $4 - 3i$ .

complex roots  
come in  
conjugate pairs

$$4 + 3i$$

sum: 8

product: 25

$$\boxed{x^2 - 8x + 25 = 0}$$

27) Write the equation of the quadratic given that one root is  $5 + 2i$ .

sum: 10

$$5 - 2i$$

product: 29

$$\boxed{x^2 - 10x + 29 = 0}$$

5	25	-10i
2i	10i	-4i^2

28) Solve the polynomial equation for all zeros:  $2x^3 + 14x^2 + 20x = 0$

$$2x(x^2 + 7x + 10) = 0$$

$$2x(x+2)(x+5) = 0$$

$$\boxed{x=0 \quad x=-2 \quad x=-5}$$

29) Determine the nature of the roots of the quadratic  $-3x^2 + 4x - 2 = 0$ .

$$\text{Discriminant} \approx b^2 - 4ac$$

$$(4)^2 - 4(-3)(-2)$$

$$-8$$

2 imaginary roots

30) Consider the polynomial function:  $f(x) = x^3 + 2x^2 - 13x + 10$

a) Show that  $x = 4$  is not a zero of the function.

b) Given  $x = 1$  is a zero, what must be a factor of  $f(x)$ ?

c) Find the remaining zeros.

d) Express the polynomial in terms of linear factors.

a)  $f(4) = (4)^3 + 2(4)^2 - 13(4) + 10 = 54$

There is a remainder  
of 54

b)  $(x-1)$  is a factor of  $f(x)$

c) 
$$\begin{array}{r|rrrr} 1 & 1 & 2 & -13 & 10 \\ \downarrow & 1 & 3 & -10 & \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

d)  $f(x) = (x-1)(x+5)(x-2)$

$$x^2 + 3x - 10$$

$$(x+5)(x-2) = 0$$

$x = -5 \quad x = 2$

